ISSN 2229-5518

On the Dual Real Value Nature of Complex Numbers

P. Harsha

Abstract— Since its inception, complex numbers remain without a proper mathematical value i.e. they cannot be designated a position on the number line. The paper gives a hypothesis that can give complex numbers a real (mathematical) value. The research work investigates how a complex number behaves in terms of real numbers thereby finding a way to give complex numbers a real value. Though the hypothesis may seem unacceptable its mathematical and physical significances, discussed in the paper, are vindicative of such an answer. The very fact that a complex number can be given a real value can prove to be useful especially in the field of complex analysis.

Index Terms— complex number, imaginary value, real value, square root, duality, number line, Cauchy-Riemann equations, electromagnetism, .

1 Introduction

A complex number is a number which can be put in the form a + i b, where a and b are real numbers and i is called the imaginary unit. Mathematically i = √–1 [1]. Italian mathematician Gerolamo Cardano introduced complex numbers and he called them "fictitious", during his attempts to find solutions to cubic equations in the 16th century [2]. Complex numbers are still imaginary numbers, despite the huge advancements in mathematics. Many mathematicians in the past years have tried to give a quantitative value to complex numbers but none could give 'i' a comprehensive value. Complex numbers don't have any real value which means they cannot be placed anywhere on the number line. For e.g. we consider the number 'pi' which has a value 3.14... and the decimals keep going. Though this number's decimals aren't definite we have a specific idea about where it would be on the number line. But if we consider a complex number 3 + i 5, there is absolutely no way to have any idea of where that number would be on the number line. I have given a solution for the real value nature of complex numbers in my research work and I have found a way by which we can assign real values to a complex number, which technically has no definite value. By doing so, it unexpectedly brings the property of duality to mathematics. Duality is a famous concept in physics-wave-matter duality etc. But such a concept has never existed in mathematics. According to mathematics, an entity can have only one value or one value among two or more possible values. Mathematics doesn't permit any variable to "hold" more than one value. My hypothesis proposes that complex numbers have a dual value nature in terms of real numbers, thus the title Dual Real Value Nature of Complex Numbers. I have also mathematically and physically verified the possibility of such an answer in the paper. The fact that complex numbers can be expressed in terms of real values could prove to be advantageous for future mathematics.

• *P. Harsha is currently pursuing bachelor degree program in electronics & communication engineering at Chennai Institute of Technology, India, PH-+919791054099. E-mail: hp07@india.com*

2 The Hypothesis for the Value of 'i'

By mathematical definition,

$$i^2 = -1*1 \quad [2]$$

From the expression,

$$i*i = -1*1$$

It can be noticed that,

$$i = +1 \text{ AND } i = -1$$

This would be the only way to equate the expression and it implies that '*i*' takes both values of -1 and +1. This does indeed contradict the fundamentals of mathematics.

Thus if we take a complex number 3 + i 5, it would have both real values of -2 AND 8. This is how a complex number would behave in terms of real numbers.

Mathematics has no provision for such an answer. Mathematics abides by a principle of one variable being capable of holding only one value. Even in the case of conventional square roots, a variable can have only one of the two values, either the positive answer OR the negative answer. The reason why I have stressed 'or' will be explained later in the paper. And further more, computers too work only on this principle. A single memory box can't hold more than one value.

This duality principle proposed at this time can be compared to the opposition that negative numbers received when they were introduced. At those times, the concept of a number being negative was simply not conceivable. But today negative numbers are as important as the positive ones, in not only mathematics but also in physics, chemistry and other sciences. Mathematical duality that I propose also is the same case. Though it seems unacceptable for an enitity to have more than one value, it is the only way to answer a question that even the most modern mathematics principles can't answer i.e. giving complex numbers a "conceivable" value. This is the hypothesis I wish to state and it totally betrays mathematics because now a variable can hold two values (if that variable is assigned to a complex number). Though I would love to prove this hypothesis, I regret that I am unable to give a concrete proof to such an absurd answer. But to show the scientific community the validity of my hypothesis I have discussed some proofs that show that the above answer does exist.

3 Analysis of Square Roots

The square root of a positive number is well known and well defined.

Eg: $\sqrt{25} = \pm 5$; In words, Square root of 25 is + OR – 5.

But using the result discussed in the paper, the square root of a negative number can be found.

Eg: $\sqrt{-25} =$ $\oplus$ 5; In words, Square root of -25 is + AND – 5.

The $\oplus$ symbol is just to emphasize AND relation of the roots while ± emphasizes OR relation between the roots.

Square root of a positive number gives a + OR – answer. But, Square root of a negative number gives a + AND – answer.

Mathematically,

$$x = \sqrt{-25} = (5) \wedge (-5) \quad ; \quad x = \sqrt{25} = (5) \vee (-5)$$

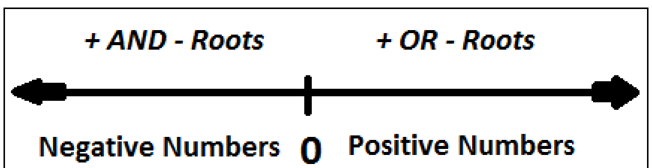

Fig 1. Symmetry of square roots on the number line

On closing looking at this analysis of the number line, we can visualize the symmetry of arrangement. The difference in the answers of the two square roots is basically caused by the relationship between the roots – Conjunction or Disjunction. But the AND-OR relation might not have any direct implications from Boolean algebra or set language, if there are such implications then that would be the proof to my hypothesis.

This arrangement is a testament to the hypothesis but on further research, this result *has direct consequences in physics and its existence can be mathematically shown* as said before.

4 Mathematical Proof of Existence

Cauchy-Riemann Equations [3]: The differential equations give the necessary condition for a complex function f(z) to be regular.

If w = f(z), where w = u + *i* v and z = x + *i* y and since u and v are both functions of x and y and therefore we can write

$$w = f(z) = u(x,y) + i\, v(x,y)$$

Now if w is differentiable at a given point z, the limiting value must tend to a certain finite limit as Δz →0 from any direction.

$$\Delta z = \Delta x + i\, \Delta y$$

If Δz is wholly real then Δy = 0, differentiating w with respect to x

$$\frac{dw}{dx} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

Similarly again if Δz is taken wholly imaginary then Δx = 0 and we get the limiting value again by differentiating w with respect to y.
We get

$$\frac{dw}{dy} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$$

Till the computation of the two limiting values, the derivation is same. The subsequent steps are

a) continues as per the conventional derivation
b) uses the hypothesis discussed in the paper

4.1 Regular Derivation

Since the function is differentiable, the two limiting values so obtained must be identical

$$\frac{dw}{dx} \equiv \frac{dw}{dy}$$

Equating real and imaginary parts the Cauchy-Riemann Equations are obtained

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \;\&\; \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

4.2 Hypothesis Based Derivation

The following derivation is extremely tedious compared to the conventional derivation. But it can be argued that my hypothesis is correct if I can arive at the Cauchy-Riemann equations using my hypothesis in the derivation.

In $\frac{dw}{dx} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ and $\frac{dw}{dy} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$ taking the '*i*' and replacing it with $\oplus$ 1 according to the hypothesis in the paper we get,

$$\frac{dw}{dx} = \frac{\partial u}{\partial x} \oplus \frac{\partial v}{\partial x} \;\&\; \frac{dw}{dy} = \frac{\partial v}{\partial y} \oplus \frac{\partial u}{\partial y}$$

ISSN 2229-5518

Since it's a $\oplus$ sign before $\frac{\partial u}{\partial y}$ the '–'sign becomes meaningless like that in '±'.

Now equating the two limiting values ($\frac{dw}{dx} \equiv \frac{dw}{dy}$) taking into account the AND relation four equations can be obtained.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} + \frac{\partial u}{\partial y} \quad (1); \qquad \frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} + \frac{\partial u}{\partial y} \quad (2);$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \quad (3); \qquad \frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \quad (4);$$

Adding (1) & (2)

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} + \frac{\partial u}{\partial y} \quad (5);$$

Adding (1) & (3)

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} \quad (6);$$

Adding (1) & (4)

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad (7);$$

Adding (2) & (3)

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad (8);$$

Adding (2) & (4)

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} \quad (9);$$

Adding (3) & (4)

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \quad (10);$$

Subtraction is not done since it involves a change in sign of one of the equations alone, disrupting the basic relationship between the equations because each equation isn't a separate entity but only based on the dual nature (AND principle). It can be observed equations (1) (2) (3) (4) are the identical but for the signs in between them.

Equations (7) and (8) are the same (again proving AND relation between the subsequent equations as well, though the equations look different they are actually the same) and are one of the Cauchy-Riemann equations.

Equating (5) & (6)

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial y} = \frac{\partial v}{\partial y} - \frac{\partial v}{\partial x} \quad => \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad (12);$$

Similarly equating (9) & (10)

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \quad => \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad (13);$$

Equations (12) and (13) are the same and are the other Cauchy-Riemann equation.

As a result both the required differential equations are obtained, but on differently equating (5) & (10) using the converse of the hypothesis (replacing the corresponding + AND – real equations into a single complex equation with the imaginary part 'i')

We get,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} + i\frac{\partial u}{\partial y}$$

Similar equating of (6) & (9) we get

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

The mathematical significance of these two equations is that if those two equations are equated again and simplified further, we get the second Cauchy-Riemann equation

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

It's like a Round-Robin process whereby further and further combinations of equations and their subsequent simplifications we only end up with the two basic differential equations. *Though this method is a tedious (and a difficult to understand) way to derive the Cauchy- Riemann equations, it proves mathematically the existence of AND relation in complex numbers, functions and variables, as the two basic differential equations are indeed finally obtained.*

5 Physical Proof of Existence

Consider a straight wire carrying steady current along z-axis. The magnetic field is along the plane (x-y plane) perpendicular to the wire (along z-axis). The electromagnetic field is a complex number [4]. Since the electric field is steady and unidirectional through the wire, it becomes the real part of the complex number - electromagnetic field. So naturally the magnetic field becomes the imaginary part of the complex number.

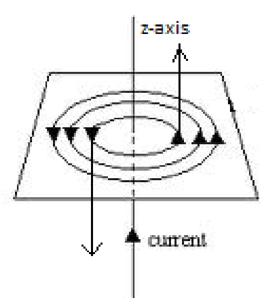

Fig. 2 Magnetic field due to a Straight wire carrying field

As depicted in the diagram the two lines are drawn on the plane of the magnetic field and they indicate the direction of the magnetic field at the two respective points. At those points, the magnitude of the field is the same but they are opposite to each other in direction. Since magnetic field is a vector it means that simply one is the negative of the other. And the key point to note here is that *the field exists in such a way and in such a direction at the two points at the same time, indicating the + AND – relation in the value of i (magnetic field is considered as the imaginary part).*

This property is not restricted to the two points in consideration but for any point in a magnetic field due to a straight wire carrying current there will always be another point where the magnitude of the field is same but the direction of the field is exactly opposite to the direction of field at the former point. This is a direct consequence of the hypothesis developed

Now why is the magnetic field circular and not just along those two opposite directions? It's a question of probabilities. Now let us assume the field to be circular.

There can be 'p' number of orientations possible along the circumference of the circle for those two diametrically opposite field lines. Now the probability that those two field lines will exist in one of 'p' number of orientations is 1/ p.

$$\alpha = p * \phi$$

'α' is probability of field being circular, 'ϕ' is probability of two diametrically opposite field lines being present in any one of the orientations and 'p' is total number of possible orientations which is actually a large number.

Probability of field being circular = p * (1/ p) = 1

This shows mathematically that the field has to be circular in nature. Concentric circles of magnetic field are generated due to the change in the magnitude of the magnetic field. It must be remembered that a complex number a + ib has a real part and a imaginary part. But the 'b' in the imaginary part is a real number. So with respect to the above example 'i' gives the magnetic field its circular nature while 'b' determines the magnitude of magnetic field at a particular distance from the wire. Thus it can be shown mathematically why the magnetic field of a straight wire carrying current is concentric circles around the wire and this type of a field has been observed experimentally. The converse of this effect is observed in a solenoid carrying current generating a linear magnetic field.

6 Conclusion

Through this paper a real value is assigned to '*i*'. Though the result is difficult to accept, the symmetry of arrangement of square roots on the number line, the hypothesis based derivation which still leads to the same set of Cauchy-Riemann differential equations and the real life example – the electromagnetic phenomenon are evidences to the presence of such an answer. The result gives mathematics an entirely new unprecedented approach. Now it is possible for an entity to hold more than one value at the same time which contradicts basic mathematics but gives it a whole new dimension and scope. By means of the result, '*i*' can finally be mathematically valued and that as said earlier would prove to be valuable in complex analysis and other fields of mathematics too.

Acknowledgment

The author wishes to thank and dedicates his paper to his parents Dr. A.R.Prabakaran and Mrs. Lakshmi Kumari. The work wouldn't have been possible without them and their constant encouragement. The author also wishes to thank all his mathematics teachers who paved way for him to do the research work and gain sufficient knowledge and understanding of mathematics.

References

[1] Burton, David M. (1995), *The History of Mathematics* (3rd ed.), p. 294, New York: McGraw-Hill, ISBN 978-0-07-009465-9

[2] Complex Numbers – Wolfram Math world http://mathworld.wolfram.com/ComplexNumber.html

[3] BD Gupta, Mathematical Physics, Vikas Publishing House Pvt. Ltd., pp. 476-479, 1980

[4] Question Corner – Complex numbers in Real life http://www.math.toronto.edu/mathnet/questionCorner/complexinlife.html